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RAN-1901001101060001

M. A. (External) (Part - 1) Examination March - 2026

Mathematics (Paper - 406)

P. D. E. and Fourier Analysis

Time: 3 Hours] **[Total Marks: 100**

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book. (2) There are five questions in this question paper (3) Answer all questions (4) Figure to the right indicates full marks of the questions.	Student's Signature

Q. 1. (a) Obtain the direction ratios of the tangent line to a curve in space. **7**

(b) Discuss Charpit's method. **7**

(c) Solve the equation $(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$ **6**

OR

(a) Find the orthogonal trajectories on the sphere $x^2 + y^2 + z^2 = a^2$ of its intersections with the paraboloids $xy = cz$. **7**

(b) Find the integral curves of the equations and show that they are circles. **7**

(c) Describe the method to obtain orthogonal surfaces to the system $f(x, y, z) = c$. **6**

Q. 2. (a) Solve the equation $(y + z)dx + (z + x)dy + (x + y)dz = 0$ **7**

(b) Show that the differential equation is integrable and find its primitive. **7**

(c) Find the surface which intersects the surfaces of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$. **6**

OR

(a) Show that a necessary and sufficient condition for the relation $F(u,v) = 0$ is $\partial(u, v)/\partial(x, y) = 0$. **7**

(b) Find the surface which is orthogonal to the one-parameter system $z = cxy(x^2 + y^2)$ and which passes through the hyperbola $x^2 - y^2 = a^2, z = 0$. **7**

(c) If a characteristic strip contains at least one integral elementof $F(x, y, z, p, q) = 0$ then show that it is an integral strip of the equation $F(x, y, z, z_x, Z_y) = 0$. **6**

Q. 3. (a) Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the x-axis. **7**

(b) Find a complete integral of the equation $p^2y(1 + x^2) = qx^2$ **7**

(c) Determine simultaneous differential equations of first order and first degree. **6**

OR

(a) If $u_1, u_2, ..., u_n$ are solution of the homogenous linear partial differential equation $F(D, D')z = 0$ then prove that $\sum_{r=1}^{n} c_r u_r$; where the $c_r's$ are arbitrary constants, is also a solution. **7**

(b) Find the particular integral of the equation $(D^2 - D')z = e^{2x+y}$. **7**

(c) Solve the equation $r + s - 2t = e^{x+y}$. **6**

Q. 4. (a) Find the Fourier series for the function $f(x) = \left(\frac{\pi - x}{2}\right)^2$; $0 < x < 2\pi$ **7**

(b) Expand the function $f(x) = e^x$, $-\pi < x < \pi$ in terms of Fourier series. **7**

(c) Derive the complex Fourier series for the interval $[0, 2\pi]$. **6**

OR

(a) Define orthonormal system of functions. Prove that the sum of the squares of the fourier co-efficient of a square integrable function always converges. **7**

(b) Find the Fourier series to represent the function $f(x) = x^2$, f or $x \in [-\pi, \pi]$ and use Parseval's identity to show $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} .... = \frac{\pi^4}{90}$ **7**

(c) Define odd and even functions. Discuss about half range sine and cosine fourier ies. **6**

Q. 5 (a) Prove the followings: **7**

(a) Prove the followings:

3) $\frac{1}{\pi} \int_{-\pi}^{\pi} D_n(u)du = 1$; *where* $D_n(u)$ *is Dinchlet's kernel*

4) $D_n(u) = \dfrac{\sin(n + \frac{1}{2})u}{2\sin(\frac{u}{2})}$

(b) Define orthogonal system and complete orthonormal system of functions. Prove that if $\{\varnothing_n(x)\}$ be a complete orthonormal family of functions and f and g be integrable over $[a, b]$ then f = g iff fourier expansion of f is equal to the fourier expansion of g **7**

(c) Derive Fourier integral formula. **6**

OR

(a) Find the cosine transform of x defined as: $f(x) = \begin{cases} 1; 0 \le x < a \\ 0; x \ge a \end{cases}$ What is the function whose cosine transform is $\sqrt{\frac{2}{\pi}} \left(\frac{\sin ak}{k}\right)$? **7**

(b) Show that **7**

(c) $\mathcal{F}_c \{e^{-ax}\} = \sqrt{\frac{2}{\pi}} \left(\frac{a}{a^2 + k^2}\right)$; $a > 0$

(d) $\mathcal{F}_s \{e^{-ax}\} = \sqrt{\frac{2}{\pi}} \left(\frac{a}{a^2 + k^2}\right)$; $a > 0$

(c) Prove that the sum of the squares of the Fourier co-efficient of a square integrable function always converges. **6**